

# Back-Reaction of Cosmological Perturbations in the Infinite Wavelength Approximation

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Cosmological perturbations in an expanding universe back-react on the space-time in which they propagate. Calculations to lowest non-vanishing order in perturbation theory indicate that super-Hubble-scale fluctuations act as a negative and time-dependent cosmological constant and may thus lead to a dynamical relaxation mechanism for the cosmological constant. Here we present a simple model of how to understand this effect from the perspective of homogeneous and isotropic cosmology. Our analysis, however, also shows that an effective spatial curvature is induced, indicating potential problems in realizing the dynamical relaxation of the cosmological constant by means of back-reaction.

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## I. INTRODUCTION

The cosmological constant problem is one of the most important problems of theoretical physics. The issue is to find an explanation for the fact that the value of the cosmological constant in the present universe - inferred from observational upper bounds on the contribution of the cosmological constant to the “energy” content of the present universe - is much smaller than what can be deduced from theoretical estimates for the vacuum energy density  $\rho_{vac}$ . From considerations of quantum field theory one would expect the vacuum energy density to be of the order  $M^4$ , where  $M$  is the ultraviolet cutoff of the theory. In non-supersymmetric theories, the mismatch is by a factor of  $10^{-120}$  (see e.g. [1, 2] for reviews). In supersymmetric models,  $\rho_{vac}$  is of the order  $M_s^4$ , where  $M_s$  is the scale of supersymmetry breaking. Thus, the theoretical predictions still exceeds the observational bounds by about 60 orders of magnitude.

Recent data, both from supernovae observations [3] and also from cosmic microwave anisotropy measurements (see [4] for the most recent observational results) indicate that the universe is right now entering a new stage of acceleration, indicating the presence of something that acts as an effective cosmological constant with the value  $\Lambda_{eff} \sim 10^{-120} M_p^4$ . Thus, there now appear to be two aspects of the cosmological constant problem, firstly why the cosmological constant is so tiny compared to theoretical expectations (the “old cosmological constant” problem - using Weinberg’s language [5]) and the problem of why, given that it is so small, the cosmological constant does not exactly vanish, but is becoming visible

precisely at the present time of cosmic history (the “new” cosmological constant” or “coincidence” problem) .

Independent studies of the back-reaction effects of long-wavelength gravitational waves [6] and of long-wavelength cosmological perturbations [7] on the background geometry of space-time have led to a scenario [8, 9] in which these fluctuations lead to a dynamical relaxation of an initially large bare cosmological constant. The large initial cosmological constant leads to a period of primordial inflation. During this period, quantum vacuum fluctuations of both gravitons and scalar metric fluctuations are stretched beyond the Hubble radius, thus generating a large phase space of long-wavelength modes (see e.g. [10, 11] for reviews of the theory of cosmological fluctuations). In an inflationary background, it can be shown that these modes have a back-reaction effect on the local geometry which is analogous to that of a negative cosmological constant (this effect may not be physically measurable in models with only one matter field [12, 13, 14] - see, however, [15] for a different conclusion -, but it is definitely physically measurable in models with two or more matter fields [16]). This opens up the possibility that back-reaction may lead to a dynamical relaxation of the bare cosmological constant.

In this note we wish to focus on a major problem with the past analyses of the gravitational back-reaction mechanism (see e.g. [12] for a discussion of these and other problems). Namely, if the back-reaction is due to extremely long wavelength modes (for which contributions from spatial gradient terms are negligible), then it should be possible to understand the physical effect from the point of view of the local equations of homogeneous and isotropic cosmology [29].

We present a simple way of modeling the back-reaction effects of long wavelength fluctuations in terms of the equations of homogeneous and isotropic cosmology (see also [17] for an earlier work on the modeling of non-

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perturbative effects). Our result is that the back-reaction of long wavelength cosmological fluctuations in an inflationary background appears as a local fluctuation of the cosmological constant. On average, the effect leads to a *decrease* of the cosmological constant. However, in a general background, a positive correction to the spatial curvature is induced. This may lead to stringent constraints on the back-reaction mechanism.

In inflationary cosmology the phase space of super-Hubble fluctuations builds up over time since the wavelengths of modes with fixed comoving wavelength redshift relative to the Hubble radius. We incorporate this result into our local cosmological model by taking the density perturbation to increase over time, which is modeled by a decreasing cosmological constant. As discussed in the perturbative framework in [9], this leads to a *dynamical scaling solution* in which

$$\Omega_{\Lambda_{eff}}(t) \sim 1, \quad (1.1)$$

(where  $\Omega_X = \rho_X/\rho_c$  is a measure of the contribution of  $X$  to the closure density  $\rho_c$  of the universe,  $\rho_X$  denoting the effective energy density in some  $X$  “matter”) at all sufficiently late times  $t$ . Thus, this dynamical relaxation mechanism would automatically address both the old and the new cosmological constant problems.

The reason why one hopes to obtain the dynamical fixed point solution (1.1) is as follows: as the phase of inflation proceeds, the phase space of infrared modes builds up. Since long-wavelength fluctuations are frozen, the contribution of an individual mode to the effective cosmological constant does not decrease in absolute magnitude. Thus, the total back-reaction effect builds up gradually, and the effective cosmological constant, which is the sum of the positive bare cosmological constant and the induced negative back-reaction term, decreases. However, the sum cannot drop to zero, since before this happens the energy density  $\rho_\Lambda$  corresponding to the *effective* cosmological constant will drop below the matter energy density [30]. As soon as this happens, inflation will end, the phase space of infrared modes will cease to increase, and the back-reaction contribution to  $|\Lambda_{eff}|$  levels off. Since the universe is still expanding, the matter energy density  $\rho_m$  (where “matter” here stands for both cold matter and radiation) continues to decrease, thus allowing  $\rho_\Lambda$  to start dominating again, enabling the back-reaction effect to again increase in strength. Thus, as explained in detail in [9], we expect that at all sufficiently late times  $\rho_\Lambda/\rho_m$  will be oscillating in time about the value 0.5 [31].

## II. BACK-REACTION OF COSMOLOGICAL PERTURBATIONS: A BRIEF REVIEW

Both gravitational waves and cosmological fluctuations (scalar metric fluctuations) carry energy and momentum and thus affect the background in which they propagate. One way to describe this back-reaction effect (see

[22] for the initial work on the back-reaction of gravitational waves) is by defining an energy-momentum tensor which describes the effects of the fluctuations on the background geometry, to leading order in perturbation theory (the expansion parameter being the amplitude of the metric fluctuations measured in momentum space).

In the case of cosmological perturbations, the back-reaction formalism was initially developed in [7]. Small fluctuations of the metric and the matter fields about a classical homogeneous and isotropic background are introduced. The metric and matter fields including these linear perturbations (which are taken to satisfy the linear perturbation equations) are inserted into the Einstein equations. These equations are then expanded to second order. The Einstein equations are not satisfied at second order, and it is necessary to add back-reaction terms which are quadratic in  $\epsilon$  (the relative amplitude of the linear fluctuations). In particular, one needs to add a second order correction to the zero mode of the metric. The sum of the background metric plus the quadratic zero mode correction defines a new homogeneous metric  $g_{\mu\nu}^{(0,br)}$  which takes into account the effects of back-reaction to this order. This new metric obeys the modified equations

$$G_{\mu\nu}(g_{\alpha\beta}^{(0,br)}) = 3\kappa \left[ T_{\mu\nu}^{(0)} + \left\langle T_{\mu\nu}^{(2)} - \frac{1}{8\pi G} G_{\mu\nu}^{(2)} \right\rangle \right], \quad (2.2)$$

where  $\kappa = 8\pi G/3$ , the constant  $G$  denoting Newton’s gravitational constant,  $G_{\mu\nu}$  is the Einstein tensor,  $T_{\mu\nu}$  is the energy-momentum tensor of matter, the pointed brackets stand for spatial averaging [32], and the superscripts indicate the order in perturbation theory. The terms inside the pointed brackets form the *effective energy-momentum tensor*  $\tau_{\mu\nu}$  of cosmological perturbations. Note that all Fourier modes of the fluctuating fields contribute to the effective energy-momentum tensor for back-reaction: the effect of perturbations is cumulative. Thus, back-reaction can be important even if the relative magnitude of each metric fluctuation mode is small (as observations indicate) as long as there is a sufficiently large phase space of infrared modes, i.e. as long as the period of primordial inflation is sufficiently long.

As was found in [7], in a background space-time corresponding to slow-roll inflation, the terms which dominate the effective energy-momentum tensor  $\tau_{\mu\nu}$  of super-Hubble modes takes the form of a *negative cosmological constant* (see [23] for a recent discussion of next to leading terms)

$$p_{br} = -\rho_{br} \text{ with } \rho_{br} < 0, \quad (2.3)$$

where  $\rho_{br}$  and  $p_{br}$  stand for the energy density and pressure associated with  $\tau_{\mu\nu}$ . In hindsight, it is easy to understand why the back-reaction of infrared modes of cosmological perturbations acts as a negative cosmological constant. For long wavelength fluctuations, all terms in  $\tau_{\mu\nu}$  involving spatial derivatives are negligible. Since on long wavelengths the amplitude of the metric fluctuations is frozen (see e.g. [10]), and assuming that the background

matter fields are slowly rolling, all terms involving temporal derivatives of the fluctuation variables are also negligible. The only terms which survive are gravitational potential energy terms, terms which act as a cosmological constant. Since matter fluctuations produce negative potential wells, and since for infrared modes the negative gravitational energy dominates over the positive matter energy, the total effective energy density is negative.

The crucial observation is that the absolute value of  $\rho_{br}$  increases as a function of time. This is because, in an inflationary universe, due to the accelerated expansion of space, the phase space of infrared modes increases in time as wavelengths exit the Hubble radius.

Since it is the total metric and not the background metric only which determines observables, it was suggested [9] on the basis of the above results that the effective cosmological constant at time  $t$  is given by

$$\Lambda_{eff}(t) = \Lambda_0 + \rho_{br}, \quad (2.4)$$

where the second term on the right hand side is negative and has an absolute value which is increasing in time. Since mode by mode the magnitude of the square of the fluctuation amplitude is tiny, the back-reaction contribution to the effective cosmological constant is very small compared to the bare cosmological constant  $\Lambda_0$  during the early stages of inflation. However, if inflation lasts long enough (as will be the case in many inflationary models in the class of “chaotic” inflation [24]), then, before the homogeneous scalar field  $\varphi$  has ended the slow-rolling phase, the back-reaction contribution to the effective cosmological constant will cancel the initial bare value  $\Lambda_0$ . However, as described in the Introduction, the energy density associated with  $\Lambda_{eff}(t)$  can in fact never become negative, and at late times one will obtain a dynamical scaling solution in which the energy density associate with  $\Lambda_{eff}(t)$  tracks the matter energy density.

There are key conceptual and technical issues to be resolved before the above speculations can be considered to be well-founded. The problem we will focus on in the next section is the following: If the back-reaction effect is due to very long wavelength fluctuations, and the effect is to be physically measurable by a local observer, then it should be possible to understand the physics making use of only the local cosmological equations [33].

### III. LOCAL MODEL OF BACK-REACTION

We now propose a way of understanding the origin of back-reaction using the local equations of homogeneous and isotropic cosmology. We start with the metric including linear cosmological perturbations which in longitudinal gauge and for matter without anisotropic stress takes the form (in the absence of spatial curvature) [10]

$$ds^2 = (1 + 2\phi)dt^2 - a^2(t)(1 - 2\phi)d\mathbf{x}^2, \quad (3.5)$$

where  $\phi(\mathbf{x}, t)$  denotes the metric fluctuation,  $a(t)$  is the scale factor of the background cosmology,  $t$  is the back-

ground time coordinate, and  $\mathbf{x}$  are the comoving spatial coordinates. Note that for fluctuations with wavelengths larger than the Hubble radius, the dominant mode of  $\phi$  is independent of time if the equation of state of the background is constant [10].

As a first step towards applying the equations of homogeneous and isotropic cosmology to a local patch of the universe, we define a new time coordinate  $\tilde{t}$  via

$$d\tilde{t}^2 = (1 + 2\phi)dt^2. \quad (3.6)$$

The first local cosmological equation determines the Hubble expansion rate and reads

$$\left(\frac{da/d\tilde{t}}{a}\right)^2 = \kappa\tilde{\rho} - \frac{k}{a^2} + \frac{\Lambda}{3}, \quad (3.7)$$

where  $\tilde{\rho} = \rho + \delta\rho$  is the local energy density, and  $k$  is the curvature constant ( $k = 1$ ,  $k = 0$  and  $k = -1$  denoting the cases of a closed, spatially flat and open universe, respectively). The second local equation gives the acceleration

$$\frac{d^2a/d\tilde{t}^2}{a} = -\frac{\kappa}{2}(\tilde{\rho} + 3\tilde{p}) + \frac{\Lambda}{3}, \quad (3.8)$$

where  $\tilde{p} = p + \delta p$  stands for the pressure density.

We wish to apply these equations in an Inflationary universe. Since inflation typically renders the universe spatially flat, we will set the background value of  $k$  to zero. Let us ask how one can model the effects of a matter density perturbation  $\delta\rho$  and its associated metric fluctuations in terms of the above local equations. We wish to model fluctuation modes with wavelength far larger than the Hubble radius. On these scales, the dominant mode of the metric fluctuation variable  $\phi$  is time-independent.

Inserting (3.6) into (3.8), remembering that  $\phi$  is time-independent, and expanding to second order in  $\phi$ , we obtain

$$(1 - 2\phi + 4\phi^2)\frac{\ddot{a}}{a} = -\frac{\kappa}{2}(\rho + 3p\delta\rho + 3\delta p) + \frac{1}{3}\Lambda. \quad (3.9)$$

The terms linear in  $\phi$  and  $\delta\rho$  vanish upon spatial averaging, leaving us (after making use of the background equation) with the result

$$\frac{\ddot{a}}{a} = \kappa\rho + \frac{1}{3}\Lambda - 4\langle\phi^2\rangle\left(\frac{\ddot{a}}{a}\right)_0 \quad (3.10)$$

(where the pointed brackets standing for averaging and the subscript 0 indicates that the corresponding quantity is to be evaluated using the background metric) which implies that a long wavelength cosmological perturbation acts, when measured in terms of the background time coordinate  $t$ , like a local correction to the cosmological constant whose value is

$$\delta\Lambda = -12\left(\frac{\ddot{a}}{a}(t)\right)_0\langle\phi^2(t)\rangle, \quad (3.11)$$

i.e. to a reduction in the locally measured value of the cosmological constant. Note that this result does not, in

fact, depend on the equation of state of the background which we have assumed.

If we insert (3.6) into the other equation of motion (namely (3.7)), expand to second order in  $\phi$ , and make use of (3.11), then we obtain

$$\left(\frac{\dot{a}}{a}\right)^2 = \kappa\rho + \frac{1}{3}(\Lambda + \delta\Lambda) - 4 \langle \phi^2 \rangle \left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}\right]_0. \quad (3.12)$$

The final term on the right hand side of (3.12) acts like a local increase in the spatial curvature. During the phase of inflation, this term vanishes, but it no longer vanishes after inflationary reheating.

As mentioned in the Introduction, in inflationary cosmology the accelerated expansion of space leads to a continuous growth of the phase space of infrared modes. To model the local effects of the infrared modes in (3.11), we use

$$\phi^2(t, \mathbf{x}) = \int d^3\mathbf{k} |\phi(t, \mathbf{k})|^2, \quad (3.13)$$

where  $\phi(t\mathbf{k})$  is the amplitude of the fluctuation mode with wavenumber  $\mathbf{k}$ , and the integral runs over all wavelengths larger than the Hubble radius. In slow roll inflation, the spectrum of fluctuations is nearly scale-invariant [26],

$$k^3 |\phi(k)|^2 \simeq \text{const}. \quad (3.14)$$

Thus, the integral in (3.13) over the infrared modes depends logarithmically on  $k_{max}(t)$ , the comoving wavenumber corresponding to the Hubble radius. In turn, in inflationary cosmology  $k_{max}$  is increasing exponentially, and thus the integral grown linearly in time. Thus, the locally measured cosmological constant will continue to decrease.

Eventually, the energy density associated with the effective cosmological constant will drop below the energy density of the matter/radiation fluid. At this time, inflation will terminate. The phase space of infrared modes begins to decrease again, leading to a slow decrease of the integral in (3.13). More importantly, it follows from (3.11) that  $|\delta\Lambda|$  decreases in time since  $H(t)$  is decreasing. Thus, the cosmological constant will increase again until its associated energy density becomes larger than the matter energy density, after which the inflationary dynamics takes over again.

Combining the arguments in the two previous paragraphs, we see that our local analysis allows us to understand the origin of the dynamical fixed point (1.1). However, it also leads to a potential problem of the attempt to use back-reaction to relax the cosmological constant: it appears that back-reaction also generates a local spatial curvature term.

#### IV. DISCUSSION AND CONCLUSIONS

In this Note we have demonstrated that the back-reaction of long wavelength (super-Hubble) cosmological

perturbations on the locally measured background geometry can be understood in the quasi-homogeneous approximation (i.e. neglecting all spatial gradients) using the equations of homogeneous and isotropic cosmology alone.

When applied to inflationary cosmology, we have recovered the results of [7], namely that super-Hubble fluctuation modes affect the background space-time like an additional matter source with an equation of state of a negative cosmological constant whose absolute value is increasing in time because the phase space of infrared modes is increasing. This supports the conjectures of [9] that back-reaction of infrared modes leads to a dynamical relaxation of any bare cosmological constant. Note that the continued quantum generation of ultraviolet fluctuations is key to the relaxation. Taking only initial classical fluctuations into account, one would simply obtain a small renormalization of the cosmological constant (see also [27, 28] for related work).

However, from (3.12) we see that in a general background cosmology the back-reaction terms also induce a term which acts like a positive correction to the spatial curvature. This term vanishes during a phase of quasi-exponential inflation, but it does not vanish in late time cosmology. Since there are stringent observational constraints on the spatial curvature of the present universe, our mechanism may already be tightly constrained. One must keep in mind, however, that the observational constraints come from the properties of the cosmological fluctuations. Thus, before drawing definite conclusions, it is necessary to study how back-reaction effects the evolution of fluctuations.

A key issue is whether the back-reaction effect calculated here (and in the previous work of [7]) using the background time coordinate  $t$  can be physically measured (i.e. identified as a back-reaction effect by some physical clock). This issue was raised most forcefully in [12]. To answer this question, one has to calculate the back-reaction on a physical observable and express the result in terms of a physical clock such as  $\varphi$ . Work of [13, 14, 25] has shown that the effect of the leading infrared terms in the effective energy-momentum tensor are in fact not physically measurable (see, however, [15] for a differing view). However, they are physically measurable in models with an extra physical clock, e.g. a second scalar matter field  $\chi$  [16]. These conclusions also apply to our present work.

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- [30] See [18] for a recent study of the graceful exit from inflation in this scenario which provides naturally a large matter energy density at late times. Note also that in the absence of pre-inflationary matter, the relaxation process of  $\Lambda$  would evolve into a period of deflation [19], from which the exit into an expanding universe would be more involved.
- [31] Note that a similar scenario where the effective cosmological constant fluctuates and whose associated energy density always tracks that of matter emerges from the causal set approach to quantum gravity [20, 21]. Whereas in our approach the effective cosmological constant must remain positive, in the causal set approach its sign fluctuates.
- [32] It was shown in [7] that the back-reaction equation is covariant under linear space-time coordinate transformations.
- [33] See [25] for a previous approach to this problem.